

AD 644699

U.S. NAVAL AIR DEVELOPMENT CENTER
JOHNSVILLE, PENNSYLVANIA

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

10 OCT 1956

PHASE REPORT NO. 2
DEVELOPMENT OF DEFINITIONS AND LIMITS FOR
FREQUENCY TRANSIENTS AND
FREQUENCY MODULATION IN
AIRCRAFT 380- TO 420-CPS ELECTRIC SYSTEMS

BUREAU OF AERONAUTICS
TED Project No. ADC EL-52043

Reported by: *Oscar Markowitz*
O. Markowitz
Electrical Division

Approved by: *K. R. Smythe*
K. R. Smythe, Superintendent
Electrical Division

CLEARING HOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfiche		
\$ 3.00	\$.65	33 pp	as
/ ARCHIVE COPY			

A. C. Packard
A. C. Packard
Director

Distribution of this document
is unlimited.

Best Available Copy

DDC
RECEIVED
JAN 10 1967
RECEIVED
C

DOCUMENT RELEASE STATUS
NAVWEPS FORM 3900/3 (REV. 2-64)

THIS PAPER IS CHEMICALLY TREATED. NO CARBON PAPER IS REQUIRED

AD NUMBER
(To be
inserted
by DDC)

CLASSIFICATION

UNCLASSIFIED

UNCLASSIFIED WHEN ENCLOSURE IS REMOVED

ORIGINATING AGENCY

U.S. NAVAL AIR DEVELOPMENT CENTER

ENCLOSURE (REPORT TITLE)

Development of definitions
and limits for frequency transients and
frequency modulation in aircraft 380-
to 420-CPS electric systems. Phase rpt #2.

REPORT SERIES AND NUMBER

NADC-EL-5675

REPORT DATE

COG. BUWEPS CODE (For
use in BUWEPS only)

CONTRACT/PROJECT/WEPTASK NUMBER

TED Project No. ADC EL-52045

TO Administrator
Defense Documentation Center for Scientific
and Technical Information (DDC)
Bldg. #5, Cameron Station
Alexandria, Virginia 22314

SECURITY CLASSIFICATION

UNCLAS. CONF. SECRET

REPORT

Y

TITLE OR SUBJECT

Y

ABSTRACT OR SUMMARY

Y

Enclosure is forwarded for distribution subject to limitations checked below. Request AD number be inserted in space above; one copy of this letter be returned to originating activity if shown below; and two copies be sent to Chief, Bureau of Naval Weapons.

Qualified requesters may obtain copies of this report direct from DDC.

Qualified requesters may obtain copies of this report direct from DDC. Foreign announcement and dissemination of this report by DDC is not authorized.

U.S. Government Agencies may obtain copies of this report direct from DDC. Other qualified DDC users should request through the Chief, Bureau of Naval Weapons, Washington, D.C. 20360.

U.S. Military Agencies may obtain copies of this report direct from DDC. Other qualified DDC users should request through the Chief, Bureau of Naval Weapons, Washington, D.C. 20360.

U.S. Government Agencies may obtain copies of this report direct from DDC. Other qualified DDC users should request through no source. (Not available to non-Government DDC users.)

U.S. Military Agencies may obtain copies of this report direct from DDC. Other qualified DDC users should request through no source. (Not available to non-military DDC users.)

All distribution of this report is controlled. Qualified DDC users should request through the Chief, Bureau of Naval Weapons, Washington, D.C. 20360.

X Distribution of this document is unlimited.

Commanding Officer (Tech. Lib.)
U. S. Naval Air Development Center
Johnsville, Warminster, Pa. 18974

F. J. SAUTER
By direction

SIGNATURE

TITLE

F. J. SAUTER, Administration Officer

CODE

AD

DATE

9 JAN 1967

CLASSIFICATION

UNCLASSIFIED

Chief, Bureau of Naval Weapons (DLI-302)
Navy Department
Washington, D.C. 20360

UNCLASSIFIED WHEN ENCLOSURE IS REMOVED

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

SUMMARY AND CONCLUSIONS

INTRODUCTION

In the design of present utilization equipments destined for aircraft with constant-frequency electric systems, transient frequency characteristics of the electric system are not taken into consideration because these characteristics have not been defined or their limits specified in appropriate military specifications. Thus the equipment often fails to give anticipated performance in the aircraft even though it did provide full performance under laboratory conditions. This has forced industry to set many arbitrary definitions and/or limits for transient frequency, but to date there has been little agreement as to the extent, rate, or duration of the transient frequency changes acceptable for aircraft constant frequency electric systems.

Confusion in the setting of limits for the transient frequency characteristics has been created by the lack of commercially available instrumentation to measure frequency on a transient basis. Each engineering group interested in measuring transient frequency has used its own instruments with little intercorrespondence, reference (a). This variation in instruments has been unavoidable, since the problem has not been defined sufficiently to determine the exact information desired from the instrumentation.

Also, few data are available about the characteristics of frequency modulation. Recent compatibility problems in the field have alerted the designers of electric systems that frequency modulation must be contained within definite limits, and the designers of utilization equipment must consider the presence of frequency modulation in their designs. However, compatibility cannot be assured until characteristics are defined and limits have been established.

The Bureau of Aeronautics (BUAER) established TED Project No. ADC EL-52043 by reference (b) for the U. S. Naval Air Development Center (NADEVCEN) to review Specification No. MIL-E-7894, reference (c), and to make recommendations for revisions. Among the weaknesses of this specification revealed by the study was the absence of definitions and limits for frequency transients and frequency modulation.

This phase report discusses the problem and requirements for adequate instrumentation, and establishes definitions and limits for the transient-frequency and frequency-modulation characteristics of constant-frequency aircraft electric systems. The first phase report on the project, reference (d), described the development of definitions for voltage modulation.

SUMMARY OF ACTION

A study was made of frequency transients and frequency modulation data assembled from nearly all airframe, generator and constant speed drive manufacturers. Inconsistencies of initial rates of frequency changes during frequency transients indicated the need to examine closely the instrumentation used to obtain the data.

The types of instrumentation were investigated, with the main types reproduced and operated in the NADEVCEN laboratories. Results of the investigation revealed many areas of inaccuracies and limiting factors. No attempt was made to develop instrumentation to overcome all the inherent limitations.

Distribution of this document
is unlimited.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

Data on a limited number of types of generators, regulators, drives, and inverters were obtained in the NADEVCON laboratories for both frequency transients and frequency modulation to supplement the data received from the various engineering groups interested in the subject.

From the results of the instrumentation evaluation, a set of engineering criteria was developed. The criteria when met by all instrumentation could place all data on a common base to allow correlation.

From the data obtained from other engineering groups and NADEVCON tests, definitions and limiting criteria were developed for frequency transients and frequency modulation.

CONCLUSIONS

The instrumentation evaluation emphasized the need for improved instrumentation to measure both frequency transients and frequency modulation. The improvements are basically in the area of:

1. Measuring frequency accurately from the voltage output of a generator while the measurements are kept independent of voltage fluctuations.
2. Improving the time constant so that rates of frequency change up to 500 cps/s can be recorded accurately.
3. Improving sensitivity to frequency deviation so that frequency modulation can be recorded in a readable and accurate manner.

The study of the data of frequency transients has indicated that limits can be defined in terms of envelopes grouped in overall conditions correlated to electric system actions. To be compatible with utilization equipment designer's requirements, definition of frequency modulation should be expressed in terms of rate of frequency change and total limit of modulation.

RECOMMENDATIONS

It is recommended that the instrumentation criteria proposed in this report be adopted as a basis to determine adequacy of instrumentation in the measurement of frequency transients and frequency modulation. It is considered essential that the development of instrumentation to meet all the proposed criteria be fostered.

It is recommended that the specification changes proposed in this report be included in the Specification No. MIL-E-7894. The proposed specification changes should also be considered in properly modified forms and with appropriate design considerations for inclusion in specifications for a-c generators and regulators, constant speed drives, and electric power systems.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

TABLE OF CONTENTS

	Page
SUMMARY AND CONCLUSIONS	ii
Introduction	ii
Summary of Action	ii
Conclusions	iii
Recommendations	iii
LIST OF FIGURES AND TABLES	v
DISCUSSION OF PROBLEM	1
Definition	1
Methods of Measurement	1
INSTRUMENTATION	2
DISCUSSION	12
Frequency Transients	12
Frequency Modulation	15
PROPOSED INSTRUMENTATION CRITERIA	17
SPECIFICATION CHANGES	17
REFERENCES	19
APPENDICES	
A Derivations to Determine Error of Fundamental Component of a Variable Voltage Sine Wave Clipped at a Constant Amplitude	A-1
B Derivation to Determine the Ratio of Slope of a Triangular Wave to the Slope of a Sine Wave at Its Point of Inflection, Both Waves Having the Same Time Period and Maximum Amplitude	B-1

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

LIST OF FIGURES AND TABLES

Figure	Title	Page
1	Fundamental Voltage-Frequency Relationships for a Frequency Discriminator	3
2	Frequency Sensing Circuit (Courtesy of Westinghouse Electric Corporation, Lima, Ohio)	4
3	Diagram of Instrumentation for the Measurement of Frequency Transients and Modulation	5
4	Steady State Regulation Characteristic for the Instrumentation Shown in Figure 2	6
5	Steady State Regulation Characteristic for the Instrumentation Shown in Figure 3	6
6	Fundamental Error Introduced in Clipping a Sine Wave	7
7	Transient Characteristics of the Instrumentation Shown in Figure 2	8
8	Phase Discriminator Instrumentation and Calibration	10
9	Recording of Frequency Modulation with Navy Type E-1614-1, 2500-VA Inverter	10
10	Recording of Frequency Modulation with a Navy Type E-1737, 1500-VA Inverter	11
11	Recording of Frequency Modulation with a Hydraulic Constant Speed Drive to a 40-KVA Generator	11
12	Proposed Transient Frequency Limits	18

Table

I	Electric System Compatibility Relationships for Frequency Transients Between Aircraft Electric Systems and Utilization Equipments	20
II	Utilization Equipment Compatibility Relationships for Frequency Transients Between Aircraft Electric Systems and Utilization Equipments	20

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5875

DISCUSSION OF PROBLEM

DEFINITION

In aircraft electric power systems of the so-called constant frequency types, the frequency does not remain constant but varies during quiescent operation within some frequency band, such as 380 to 420 cps, called the steady state limit. Sudden changes in frequency within and beyond this band can occur during load disturbances or drive system input disturbances. When the disturbances are transient in nature, corresponding frequency variations including the return to steady state limits due to regulation controls, are called transient frequency characteristics. Until the characteristics of all frequency variations are defined within specified limits, designers of utilization equipment cannot assure compatibility of their equipment with aircraft electric systems.

In accordance with general usage, frequency variations fall within the following three broad categories, classified by virtue of the extent, rate, and duration of frequency changes:

1. A transient frequency change, which implies a sudden change in frequency within the definition of transient, deviates a large amount and with a high rate of change. The deviation and return to steady state is within a short time, perhaps one second.
2. Frequency modulation is the cyclic and random constant or dynamic variation of frequency about a mean frequency during steady state system operation. The frequency modulation is normally within narrow frequency limits and occurs as a result of speed variations in the generator armature due to dynamic operation of drive and armature coupling. Rates of frequency changes in frequency modulation are much lower than those due to transient frequency changes. The mean frequency can change as a result of frequency drift.
3. Frequency drift, which is the slow and random variation in frequency within the steady state limits, occurs as a result of environmental or service effects on the electric power drive system. The rates of frequency change are much lower than the other categories of frequency variations.

This report deals with transient frequency changes and frequency modulation defined in 1 and 2 above.

METHODS OF MEASUREMENT

Measurement of transient frequency is accomplished generally by using a voltage analog of the deviation from a given frequency. The voltage analog is then fed to a recorder having satisfactory response time. Since the frequency transients occur as a result of disturbances which simultaneously create voltage transients, the frequency analog has to be insensitive to voltage changes over extremely wide limits, even though the voltage is used to provide the frequency information. Examination of various instruments used in the past have indicated that none are sufficiently insensitive to voltage change to give accurate indication of the frequency changes for all conditions of electric system operation.

Many utilization equipment designers have indicated that their equipment is affected by the degree of transient frequency excursion and also by the rate of frequency change during the excursion. Thus, it becomes important to examine this rate characteristic of

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

frequency transients and contain it within specified limits. Since information about rate of frequency change has not been required from the instruments used to provide transient frequency recordings, such instruments vary in their ability to show high rates of frequency change.

Another method used to measure frequency deviations is to make direct tachometer connection to the generator drive equipment. The tachometer voltage output is a function of speed, allowing the output voltage to be calibrated in terms of frequency. This method cannot be used universally since tachometers cannot be mounted easily on drives or generators except for some laboratory operations. This method also has other disadvantages in that data involving small frequency deviations are difficult to obtain and generator transient flux shifts and shaft torsions are not seen. Based on these inherent limitations, this type of instrumentation was not included in this project investigation.

Considerable data have been published showing oscillograms of frequency transients in constant frequency systems. The data, when assembled and integrated, could provide a basis for the preparation of defined limits for the transient frequency characteristics. However, the data cannot be considered valid until the instrumentation for each set of data is verified in the light of projected definitions and limits for the frequency transients.

There are not enough data on frequency modulation to provide a firm basis for establishing limits of frequency modulation or for establishing definite design criteria which could be used with component and system design. Limited tests to obtain data on frequency modulation were made at NADEVCON to establish a starting base for developing the desired definitions and limits.

INSTRUMENTATION

Because of the diverse instruments used to measure frequency transients and frequency modulation, obtained data have not been amenable for correlation. By bringing to light the variations and inaccuracies of present instrumentation and in establishing specific requirements for all future instruments, it is hoped that the future will provide consistent and accurate data for correlation.

The approach to measuring frequency transients has been to use a frequency discriminator centered at 400 cps to obtain a voltage analog of the frequency change from 400 cps. There has been sufficient experience in the use of frequency discriminators in general and for 400 cps in particular so that it should not be difficult to obtain discriminators with good linearity through reasonable frequency changes. However, the discriminator by itself cannot provide the complete picture of transient frequency changes.

Examination of composite characteristics of aircraft constant speed electric systems has revealed:

1. The steady state operating frequency is usually near 400 cps and may be anywhere in the band between 380 to 420 cps.
2. Frequency transients may travel for the most extreme conditions to 320 or 480 cps. This represents a 20 percent change from a 400-cps base.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

3. Disturbances which cause transient frequencies also cause simultaneous voltage transients. The conditions which cause the widest extremes in frequency transients generally also cause the widest extremes in voltage transients.

4. Rates of frequency change during frequency transients may be high, perhaps as much as 500 cps/s.

These are the system characteristics which should be used as the bases for instrumentation requirements in the measurement of the frequency transients.

During extreme faults in the electric system, system voltage available for the measurement of frequency may drop to as low as 5 V, and during recovery of the fault the voltage transient may rise as high as 200 V. These are line-to-neutral rms values. Any frequency measuring instrument which uses line voltage as input should maintain its output proportional to frequency change but remain unaffected by the wide input voltage swings. This is a severe requirement not met completely by known instrumentation.

The usual varieties of basic frequency discriminators have an output proportional to both input voltage and input frequency. Figure 1 shows the basic relationships. It is apparent that interpretation of the output becomes difficult and inaccurate when the input voltage and frequency vary. In addition, when the frequency varies at rates which cannot be followed by the instrumentation, interpretation of the true frequency variation from the output becomes impracticable. To maintain the output of the frequency discriminator as a true analog of frequency variation, there must be some form of voltage regulation or limiting between the discriminator and its input.

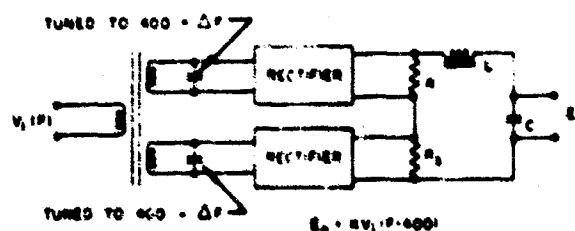
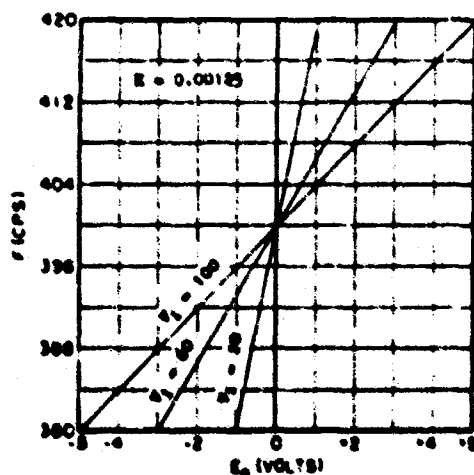


FIGURE 1 - Fundamental Voltage-Frequency Relationships for a Frequency Discriminator

Figure 2 shows a unit which uses voltage regulation of gas discharge tubes to maintain a constant voltage to the discriminator. Figure 3 shows a unit which uses electronic limiting to maintain constant voltage to the discriminator. The range of voltage regulation of the first unit (figure 2) on a steady state basis is from 54 to 150 V. The voltage-frequency characteristics are shown in figure 4. The voltage limiting of the second unit

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

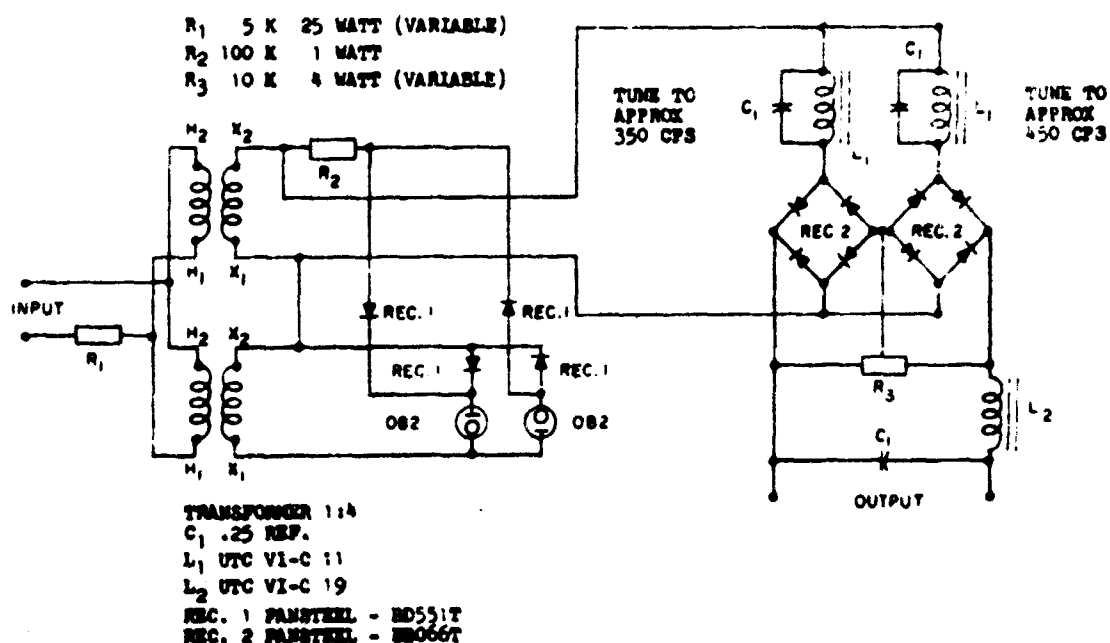


FIGURE 2 - Frequency Sensing Circuit (Courtesy of Westinghouse Electric Corporation, Lima, Ohio)

(figure 3) is fairly good down to 10 V on the ± 5 -cps span but is poor for the ± 25 -cps span. The characteristics are shown in figure 5.

Use of figure 4 can be illustrated by assuming a fault test where frequency is measured when the line voltage has been reduced to 60 V. If an oscillograph deflection of -0.6 inch is assumed, corresponding to point A on the deflection curve, this point can be moved directly across until it intersects the 60-V curve at point B, which falls on the frequency scale at 386.75 cps (line BC). If the instrumentation were assumed to be independent of voltage, and only the 115-V curve were used, then the intersection would occur at D, allowing a frequency error of 2.25 cps.

All the forms of limiting are accomplished by clipping the sine wave to provide a constant peak voltage. This clipping process introduces large and variable harmonic contents as a function of the input voltage. The peak will remain relatively constant, but there is no assurance that the fundamental content of the clipped wave will remain constant unless the voltage at which clipping begins is sufficiently below the lowest input voltage where accurate readings are required.

REPORT NO. NADC-EL-5675

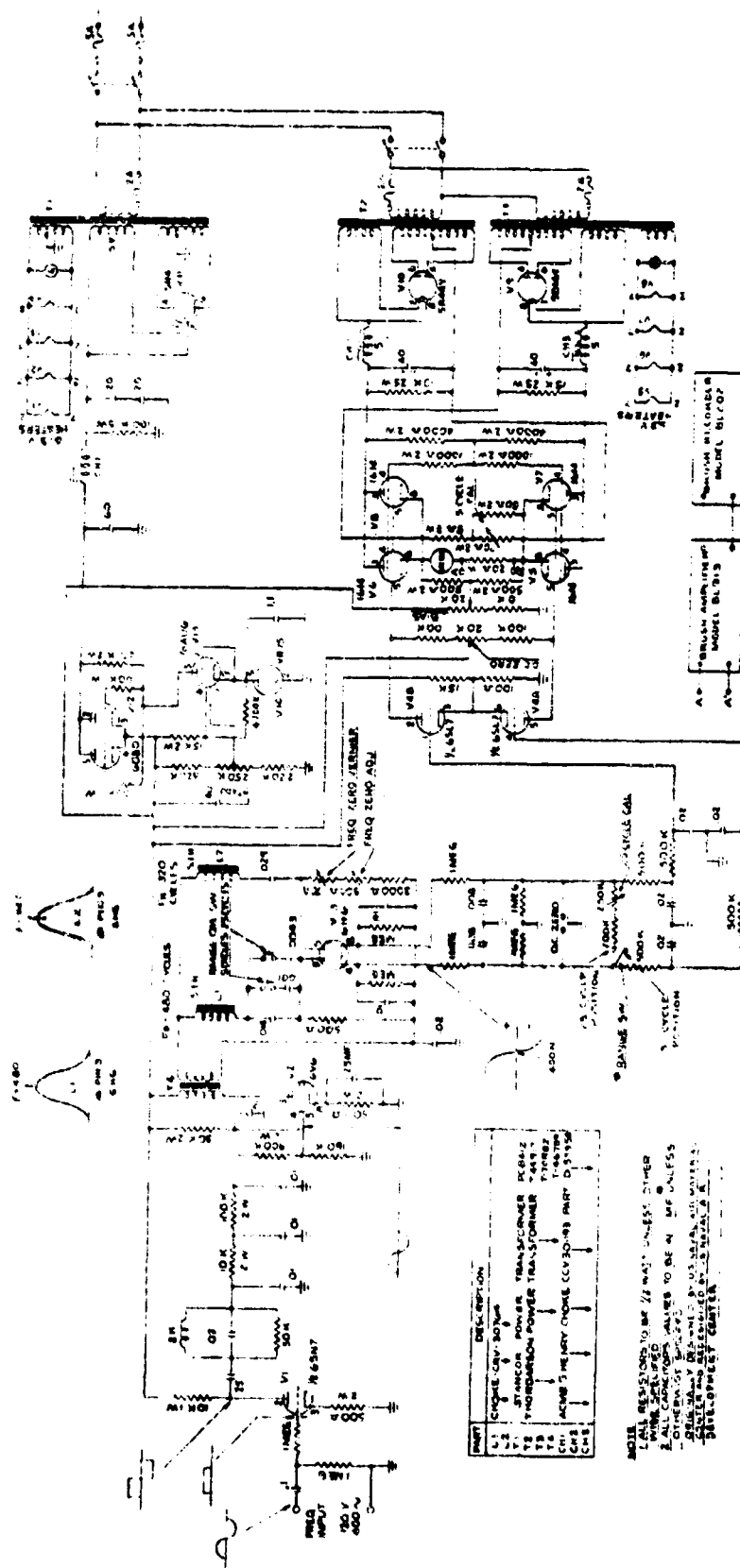


FIGURE 3 - Diagram of Instrumentation for the Measurement of Frequency Transients and Modulation

REPORT NO. NADC-EL-5675

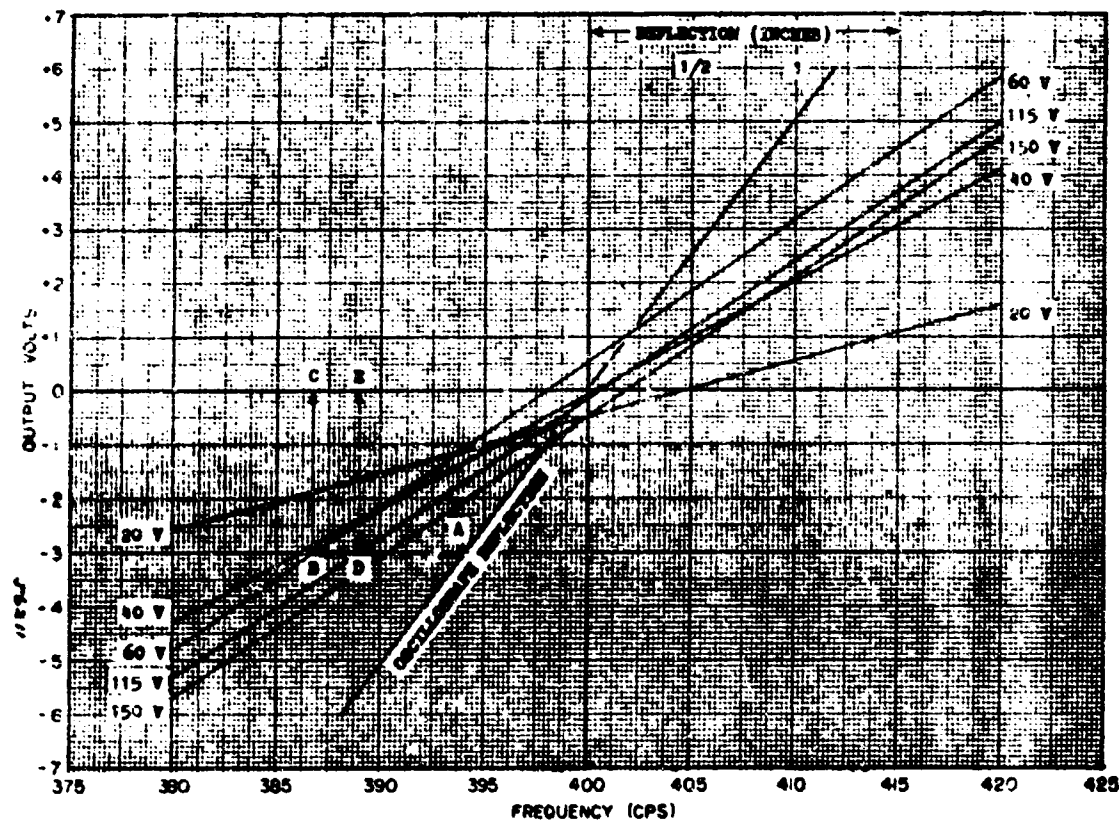


FIGURE 4 - Steady State Regulation Characteristic for the Instrumentation Shown in Figure 2

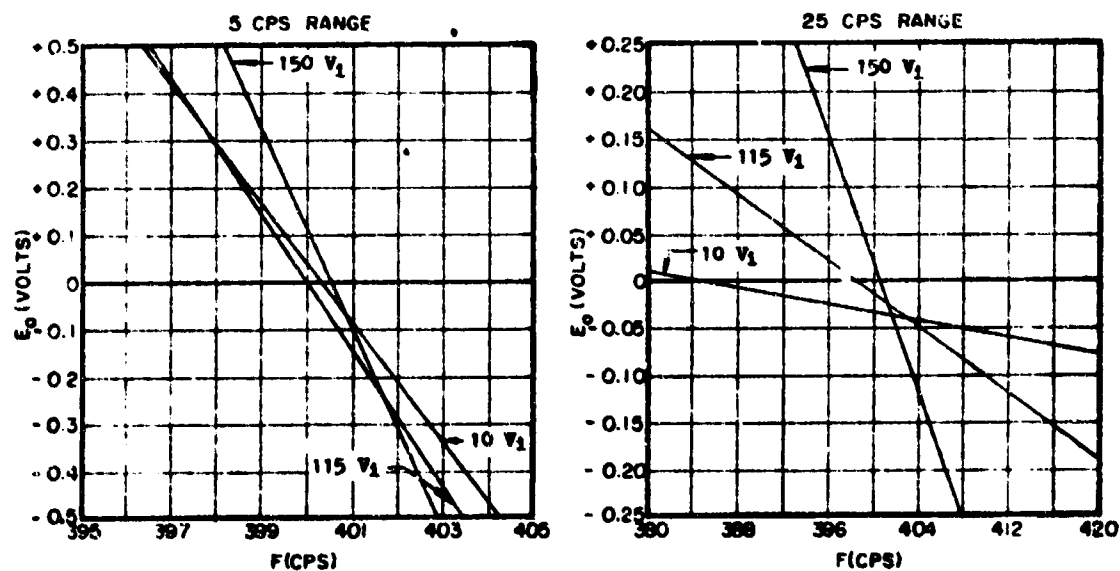


FIGURE 5 - Steady State Regulation Characteristic for the Instrumentation Shown in Figure 3

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

In appendix A a mathematical analysis is shown for the derivation of a generalization of sine wave component for the first term of a Fourier analysis. This generalization shows the varying first term as a function of the varying input voltage E_m with the clipping level remaining constant. Figure 6 is a plot of error from the derived generalization:

$$A_1 = \frac{2 E_m}{\pi} \arcsin\left(\frac{b}{E_m}\right) + \frac{2b\sqrt{E_m^2 - b^2}}{\pi E_m}$$

above 1 per unit (P.U. = $\frac{b}{E_m}$). Below 1 per unit, there is no clipping, and A_1 follows E_m down linearly to the lowest point considered (0.4 per unit), also shown in figure 6.

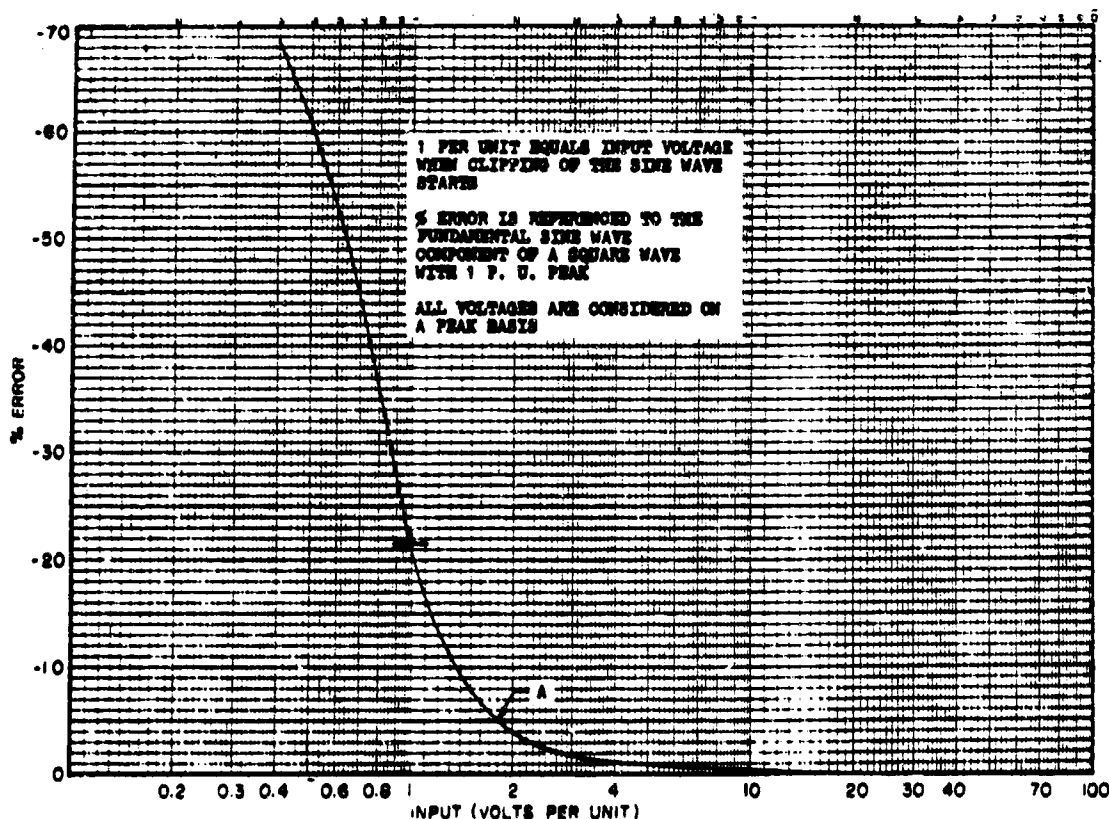


FIGURE 6 - Fundamental Error Introduced in Clipping a Sine Wave

For example, if 5 percent were allowed as maximum error and a 20-V peak was considered the lowest voltage, figure 6 may be applied as follows:

1. Point A is the 5 percent error level and correlates to 1.8 per unit.
2. Since 1.8 per unit represents the lowest voltage level for 5 percent, it corresponds to the 20 V. Therefore, 1 per unit equals 20 divided by 1.8, or 11.1 V.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

3. The resultant 11.1 V is the voltage at which clipping would have to begin to maintain a minimum of 5 percent accuracy (factor due to clipping only) at the assumed low-voltage operating point.

The instrumentation discriminator outputs are rectified a-c from the frequency responsive circuits. Ripple content in this output is generally filtered to provide clean d-c output. The filter can be of the RC or LC variety. If it is of the LC type, care must be exercised to assure sufficient loading for at least critical damping, otherwise sudden changes in input introduce a transient into the output not representing an actual frequency change. The unit in figure 2 has an LC filter and when its output is fed to a Brush amplifier-recorder, which has negligible loading, a sudden change in input frequency (or voltage) may introduce distortions in the recording of a frequency. This unit can be critically damped with a load of 12,500 ohms. In the output circuits which have RC filters, critical dampening is not a consideration.

Dynamic characteristics of the frequency discriminator are controlled to a large degree by the type and time constant of its output filter. Since the output is used with some recording means to obtain a record of frequency transients, then the dynamic action considerations of the frequency discriminator should also include the time constant of the recorder. Consideration of the dynamic characteristic is important, since it determines the maximum rate of frequency change that the instrumentation can follow accurately. The voltage analog of frequency, after being affected by the aforementioned time constants, should be capable of indicating maximum rates of frequency change of at least 500 cps/s with an accuracy better than 95 percent. This can be interpreted to mean that the slope obtained by using 3 times the time constant (TC) of the instrumentation as the ordinate and the given voltage output in terms of its frequency analog as the abscissa, defines the maximum rate of frequency change reproduced within an accuracy of 95 percent.

Figure 7 illustrates such considerations with the frequency scale spread used with the figure 2 discriminator in combination with a Brush model BL-913 amplifier and Brush model BL-202 recorder. Reproduced rates of frequency change of 500 cps/s were within the 95 percent accuracy.

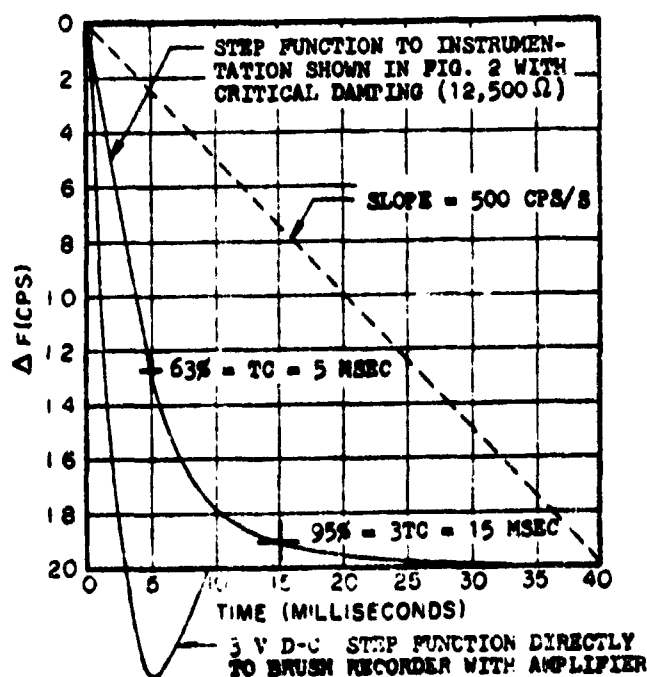


FIGURE 7 - Transient Characteristics of the Instrumentation Shown in Figure 2

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

It is apparent that by increasing the frequency scale span, higher rates of frequency change can be reproduced with a given set of equipment. This is consistent with the general requirements, since the larger rates of frequency changes are experienced during the transients which also result in the largest frequency swings. The instrumentation shown in figure 3 has a discriminator output filter which does not allow for accurate reproduction at the required high rates.

To summarize, instrumentation for recording frequency transients in 380- to 420-cps systems should have the following characteristics:

- 1 It should provide a true analog of frequency change relatively independent of any signal input voltage change between 5 and 200 V rms. This voltage range might be extended from 3 to 200 V for added facility while frequency transients are checked during extreme faults closer to the point of fault.
- 2 It should provide the frequency change information in the form of positive and negative outputs representing the frequency variations above and below a nominal frequency as zero output. The accuracy of any final frequency steady state deviation from nominal should be within 2 percent. For ease in reading a recording, more than one range in the calibrated frequency spans might be used. Suggested ranges are ± 4 , ± 20 , and ± 80 cps. It would also be helpful if the center frequency, nominally considered as 400 cps, could be readily shifted in a calibrated manner within the band between 380 and 420 cps while any given frequency span is maintained.
3. It should be capable of reproducing rates of frequency change up to 500 cps/s within a 5 percent accuracy. It would be acceptable for any full scale span of ± 20 cps or less to have at least 5 percent accuracy with rates of frequency change up to 125 cps/s.
4. The frequency discriminator portion of the instrumentation should be capable of a ± 5 -V output for each frequency span with an internal impedance of not more than 5000 ohms. This would allow reasonable matching of the output to most commercial oscillographs. Also this part of the instrumentation should have at least critical dampening regardless of what recording means are used.

Frequency modulation can be measured by the use of the equipment previously discussed. This equipment has its limitations for this purpose in that the frequency modulation for all normal conditions consists of small frequency deviations and represents only 5 to 25 percent of full-scale deflection on a sensitive range such as ± 4 cps. Thus, the same reasonable levels of interference possibly present during measurement of frequency transients might blank useful information during measurement for frequency modulation. If span sensitivity can be increased without increasing the minimum levels of interference correspondingly, and "ringing" can be avoided with increase of "Q" in the discriminator tuned networks, then a span of ± 2 cps would be adequate for direct measurement of frequency modulation.

Another approach for measurement of frequency modulation is to obtain a voltage analog of phase difference between the measured frequency and an arbitrary frequency standard. The standard would have to be maintained accurately and conveniently to within 1/20 cps of the mean of the measured frequency for the duration of the measurement. This allows full scale operation of the instrumentation for small values of frequency modulation; interference levels are very low because of inherent rejection of noise by a phase demodulator. This scheme is compromised in flexibility of measurement by the standard having a fixed frequency which cannot be varied or a variable frequency which cannot be varied in small enough increments.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

Phase discriminators or demodulators of various types as shown in references (e) and (f) can be applied to this approach in instrumentation. One arrangement as set up in the NADEVEN laboratories, and its calibration, are shown in figure 8. A phase shifter used for the calibration is described in reference (f). Recordings made with this instrumentation are shown in figures 9, 10, and 11.

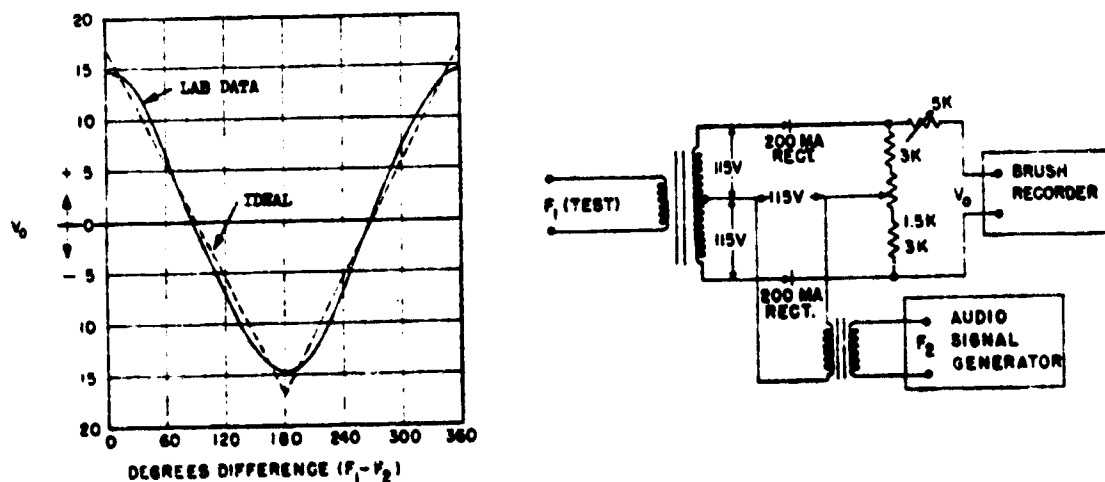


FIGURE 8 - Phase Discriminator Instrumentation and Calibration

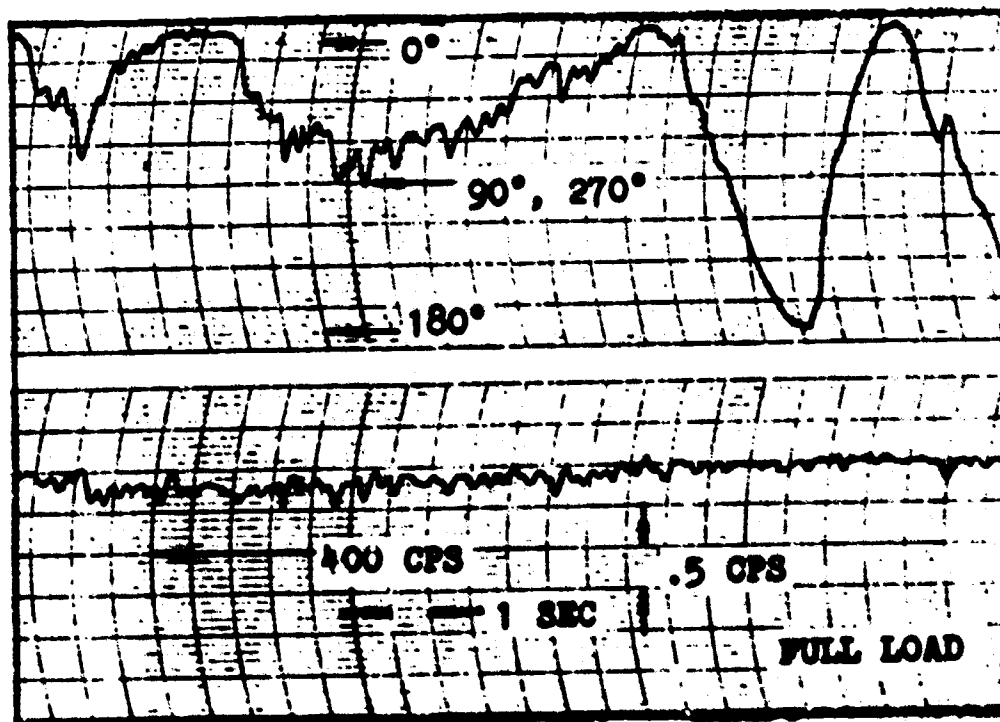


FIGURE 9 - Recording of Frequency Modulation with Navy Type E-1614-1, 2500-VA Inverter

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

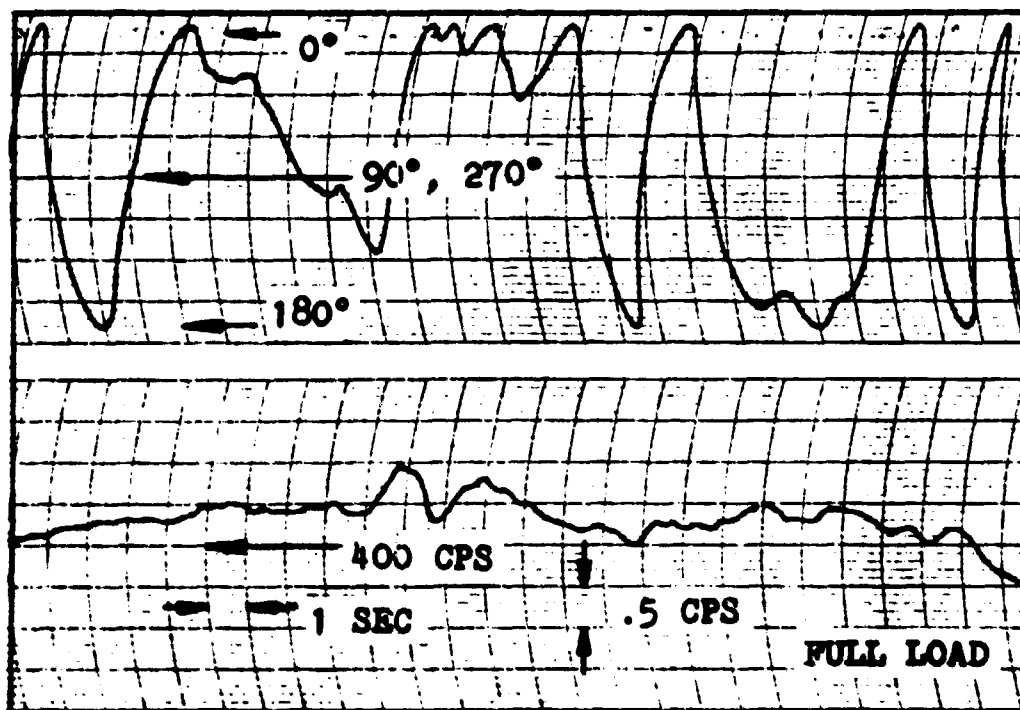


FIGURE 10 - Recording of Frequency Modulation with a Navy Type E-1737, 1500-VA Inverter

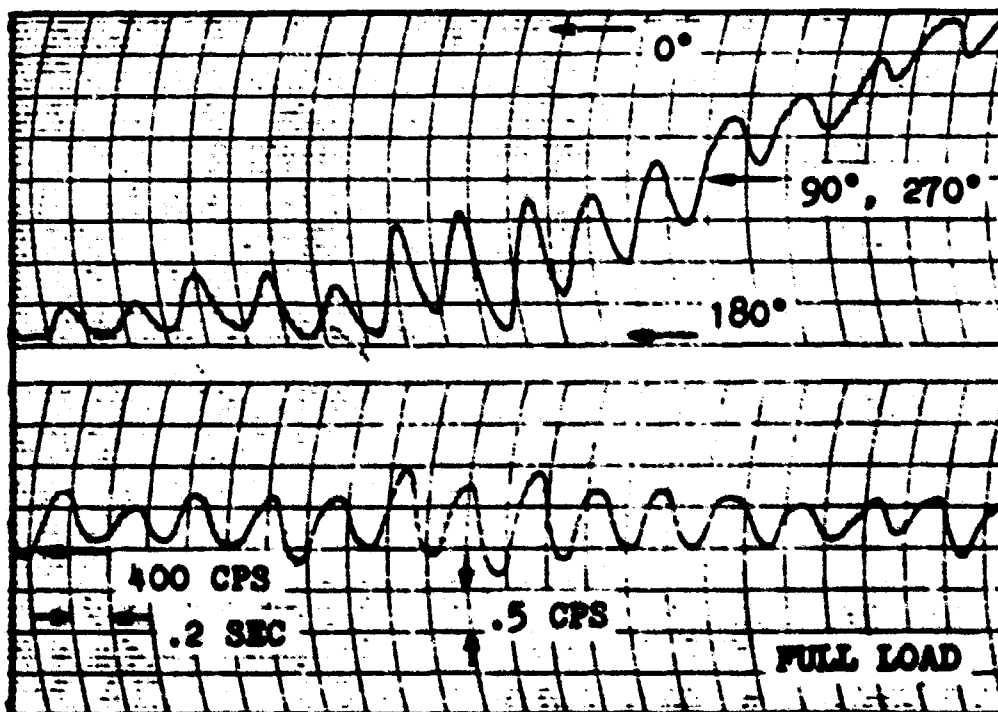


FIGURE 11 - Recording of Frequency Modulation with a Hydraulic Constant Speed Drive to a 40-KVA Generator

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

Other considerations with this type of instrumentation are as follows:

1. Direction sense of measured frequency is lost since output is plus or minus only in accordance with the relative instantaneous phase difference between measured and standard frequency. There is no direct, calibrated, way of knowing whether the measured frequency is above or below the standard frequency.
2. There is an inherent degree of voltage insensitivity. If the standard frequency input voltage is sufficiently high, then voltage changes of the measured frequency through a 2 to 1 range cause little change in output. Since frequency modulation is measured during steady state system operation, this inherent voltage insensitivity is more than adequate to keep the instrumentation independent of normal voltage variations.
3. The data are presented in the form of phase difference (ϕ) and must be interpreted to obtain frequency deviation and rate of frequency change. Slope of the data is $\frac{d\phi}{dt}$, which is equal to frequency difference from the standard with no direction sense. This data cannot be readily converted to rate of frequency change, since the second derivative $\frac{d^2\phi}{dt^2}$ is required. The second derivative is extremely difficult to obtain experimentally from the complex wave of phase deviation normally obtained. Some engineering groups have taken frequency modulation data simultaneously with both the frequency discriminator and phase discriminator types of instrumentation; the phase discriminator gives accurately small frequency differences, and the frequency discriminator gives sense of frequency change, rate of frequency change, and the larger frequency deviations.

DISCUSSION

FREQUENCY TRANSIENTS

Transients within aircraft electric systems are considered by utilization equipment designers on the basis of the extent and rate of transient excursions and reoccurrence within one equipment or aircraft operating period. Different design considerations can be made for the transients depending upon whether they occur rarely or often. Possible design correlation between the electric system and utilization equipment becomes even more apparent when it is considered that the extreme transients are those which occur rarely and only as a result of faulty system operation, and the smaller transients occur often as a result of all the normal operations of the system. Classed as a group between the two conditions of rare and normal operations, are the occasional system operations which result in large but not extreme transients.

Definitions of the transient characteristics can be made on this basis between actual systems operation and utilization design considerations. The transient limits can be defined as those within the following areas:

Area I - The first area defines transient limits resulting from all normal and usual system operations during flight preparation, airborne conditions, and landing within the required aircraft function and mission. A transient within this area could occur at any given instant, and the utilization equipment should provide 100 percent performance during and after this disturbance.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

Area II - The second area defines above-normal transients resulting from normal but extremely heavy load changes required for function of the electric system. These transients occur occasionally (three or four times) during any one aircraft operational period as a result of a controlled operation within the electric system. An aircraft operational period can be defined as the time interval from the start of preparation for flight to the postflight engine shutdown and deactivation of the electric system. These transients usually occur upon specific aircraft operations. If the utilization equipment were permitted degraded performance for the period of this transient, it should be only to the extent allowed by the detail specification for that equipment. In any case, by the end of the transient, the utilization equipment should be fully recovered to 100 percent performance with unimpaired reliability.

Area III - The third area defines the larger transients resulting from unexpected system actions. These transients rarely occur, perhaps never during the life of an aircraft, and the exact moment of occurrence is not usually anticipated. For this area of transients, correlation with utilization equipment could mean:

1. If any performance is required, it should be specified in the equipment detail specification.
2. After the transient, automatic return to 100 percent performance may not be required by the detail specification.
3. Effect on reliability should be negligible.
4. No unsafe condition should arise.

Area IV - The fourth area covers the entirely unanticipated extreme transient. The electric system designer is responsible for providing an electric system in which the transients will be confined within the specified limits of the previous three areas. In the event that a transient does go beyond the widest limit, contrary to all the design efforts, no responsibility can be expected of the utilization equipment except to remain in a safe condition and precipitate no adverse conditions other than the natural loss of the equipment function. After such a transient, the equipment would not be required to return to operation automatically. Also, when the equipment is manually returned to normal operation, reliability could be impaired.

The approach in defining the four areas places responsibility on the electric system designers without confining the individual design aspects which contribute toward making the overall characteristics.

Information on frequency transients made available to NADEVCE from four airframe manufacturers and six drive or power equipment manufacturers was evaluated in terms of the instrumentation used to obtain the data. In general, for frequency transients involving simultaneous heavy voltage swings, the data could not be considered accurate.

The curves shown in figure 12 were prepared to define the overall characteristic of frequency transients for 380- to 420-cps systems in terms of the frequency excursion limits as a function of time. The curves represent a composite frequency transient characteristic for all constant-frequency types of aircraft systems powered by the following types of constant speed drives:

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

1. Hydraulic
2. Air bleed
3. Air bleed and burn
4. Gas turbine
5. Ram air
6. Inverters of 1500-wa capacity and above
7. Drives with which paralleling is accomplished by frequency droop methods.

Compatibility of the limits in figure 8 between the electric system and utilization equipment is also considered in the four operational areas as follows and is abstracted in tables I and II:

1. Area I includes the limits of frequency excursions occurring for all normal system operations required for the functions of the aircraft. This area includes the transients created by normal connection or disconnection of loads. Thus, the area I limits connote frequency swings that would occur normally and frequently in any electric system action during any aircraft operation and can be anticipated at any time. Utilization equipment should be permitted no compromise in performance or reliability due to frequency transients within this area.

2. Area II includes the more severe limits of frequency excursion occurring during the occasional control action necessary for the electric system function, such as the initial starting of the electric system, the transferring of a power bus from one source to another, or the paralleling of generators. During these control actions, minor degradations in performance might be permitted in certain utilization equipment without compromise of overall aircraft performance. After such a frequency transient is completed, however, the utilization equipment should return automatically to 100 percent performance, with no decrease in reliability.

3. Area III is the region where the most severe transient frequency excursions occur on rare occasions during the life of an aircraft. When the frequency excursion is in this area, it would be the result of unexpected severe fault conditions within the electric system. These faults would be sufficiently serious to compromise the electric system operation during the fault, and therefore utilization equipment should also be permitted correspondingly momentary loss of performance. If the fault is cleared, the electric system should go back to normal steady state operation and utilization equipment should return to 100 percent performance, with negligible effect on reliability.

4. Area IV is the region of un contemplated transient frequency excursions. The designer uses his full capabilities to design electric systems so that they will never operate in this area. Utilization equipment should not be required to perform in this area or after the electric systems return to the other areas. In case of any remote possibility of electric system operation in this area, utilization equipment should be required to remain safe in this area and also in return to the other areas.

It is required, for proper utilization equipment design, to know the rate of frequency change during frequency transients. In many cases utilization equipment performance can

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

be influenced more by small transient frequency excursions at high rates of change than by large transient frequency excursions at low rates of change.

The available frequency transients data were reviewed to extract rates of frequency change. The rates of change during excessive voltage changes could not be considered accurate as previously discussed, but the data did indicate that the maximum rates of change for corresponding frequency transients were as follows:

<u>Operating Area</u>	<u>Rate of Change (cps/s)</u>
I	125
II	250
III	500

It is likely that frequency transients in area I have rates of change greater than 125 cps. These transients should be considered as being associated with an area corresponding to its rate of change rather than its excursion. The definitions developed should consider that the most severe aspect of the frequency transient, the rate of change or excursion, should govern in classifying its area characteristic.

FREQUENCY MODULATION

Frequency modulation originates from speed variations in a-c generator rotors due to shaft torsions and drive speed regulation dynamics used to maintain constant frequency. As long as the frequency modulation remains below reasonable levels in terms of its amplitude and rates, utilization equipment is not affected. However, it is not completely clear as to what should be considered reasonable levels. For many years 400-cps inverters have been used with considerable amplitude of frequency modulation but at unknown rates and with few recorded problems of compatibility.

The limited field experiences with compatibility problems due to frequency modulation have indicated that problems arise when the frequency modulation has gone to amplitudes above ± 5 cps. Very little data are available toward verifying that compatibility problems exist with frequency modulation amplitudes below ± 5 cps.

Two inverters and a hydraulic constant speed drive were tested at adjusted speeds and loads until the worst conditions of frequency modulation were obtained. Figures 9 and 10 show the oscillograms obtained from the inverters. Figure 11 shows the oscillogram obtained from the hydraulic constant speed drive. The instrumentation described in figure 12 was used for the upper trace and the instrumentation described in figure 3 was used for the lower trace of these recordings.

Recent developments in rate gyros operated from the aircraft electric systems have pointed toward an increased requirement to maintain limits of frequency variations within the closest practicable limits. This consideration is most applicable to frequency modulation, since the modulation represents frequency variations which occur continuously during steady-state operation at rates of change critical to many equipments.

In view of these conditions, the amplitude and the frequency characteristic (rate) of frequency modulation must be defined. A means of doing this is to use a maximum limit of ± 2 cps about a mean frequency for a given period of time to define the modulation amplitude. The ± 2 cps excursion limit for frequency modulation leads to the consideration

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

that maximum rate of change permitted for any complex wave of frequency modulation should be no worse than the maximum rate of change during a sine wave excursion of frequency modulation to the maximum allowed limits. If a continuous sine wave of frequency deviation with a 1-second period is considered as traveling to the extremes of ± 2 cps about a nominal frequency, the rates of frequency change can be averaged as a slope going from one positive extreme of the frequency deviation to the adjacent negative extreme. This slope has an ordinate of 4 cps and an abscissa of 0.5 second, giving a rate of 8 cps/s. As shown by the derivations in appendix B, there is a ratio between the slopes of the sine wave and triangular wave of 1.571 to 1 at the zero axis. Maximum slope of the sine wave occurs at its zero axis and is approximately equal to 12.6 cps/s. Thus, a maximum rate of frequency change should be set correspondingly at 13 cps/s.

The frequency characteristic of frequency modulation is generally random in its nature until one factor predominates, such as instability in a speed regulator of a constant speed drive. Under usual conditions the randomness of this characteristic does not allow ready analysis of its sine wave components but it can be limited sufficiently by specifying the maximum rate of frequency change. The maximum rate of frequency change can be identified as a maximum slope of a recording of frequency deviation, and used as a means to determine conformance of the frequency characteristic.

When a phase discriminator such as shown in figure 8 is used to record and analyze frequency modulation, correspondence must be obtained toward equivalent quantities procured on a direct frequency scale. The actual shift in degrees differentiated in respect to time $\frac{d\phi}{dt}$, is the function of frequency difference between the measured frequency and the standard frequency. The sense or direction of the frequency difference from the standard frequency is lost. A phase shift slope of 720 degrees per second corresponds to a frequency deviation of 2 cps from the standard frequency. Rates of frequency change of the measured frequency corresponds to the change from one phase shift slope to another and as such is extremely awkward to handle. Because of this difficulty of interpreting frequency data from phase shift data, it is considered that frequency rather than phase instrumentation is most applicable for the measurement of frequency modulation.

PROPOSED INSTRUMENTATION CRITERIA

It is proposed that adequate instrumentation be developed for measuring frequency transients in an aircraft constant speed electric system. Such instrumentation should tentatively provide the following overall characteristics:

1. The output (voltage analog of input frequency) should be independent of input voltage variations between the limits of 5 to 200 V rms.
2. The output should have a zero center corresponding to the nominal input frequency of 400 cps with a maximum span of ± 80 cps. A facility should be available to change the ± 80 -cps span to at least a ± 4 -cps maximum span and with an intermediate span of ± 20 cps.
3. The output should not be less than ± 5 V for full span.
4. Any interference level should be below 1 percent of any full scale output.
5. Accuracy of any steady state final output should be within 2 percent.
6. For all input frequency deviations with rates of less than 500 cps/s, the accuracy of reproduction should be within 5 percent.
7. Internal impedance looking into the output terminals should be less than 5000 ohms with critical dampening.

It is also proposed that the previously indicated instrumentation characteristics become the basic criteria for evaluating instrumentation presently in use.

SPECIFICATION CHANGES

It is proposed that Specification No. MIL-E-7894, reference (c), be expanded to include:

1. The following characteristics for frequency transients:
 - a. A frequency transient caused by usual electric system operations required directly for function and mission of aircraft shall be within area I of figure 12 and have no rate of frequency change greater than 125 cps/s.
 - b. A frequency transient caused by an occasional electric system operation required expressly for function of the electric system shall be within area II of figure 12 or less and have no rate of frequency change greater than 250 cps/s.
 - c. A frequency transient whose occurrence is not under any manual or automatic control, and is caused by a rare electric system operation, shall be within area III of figure 12 or less and have no rate of frequency change greater than 500 cps/s.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

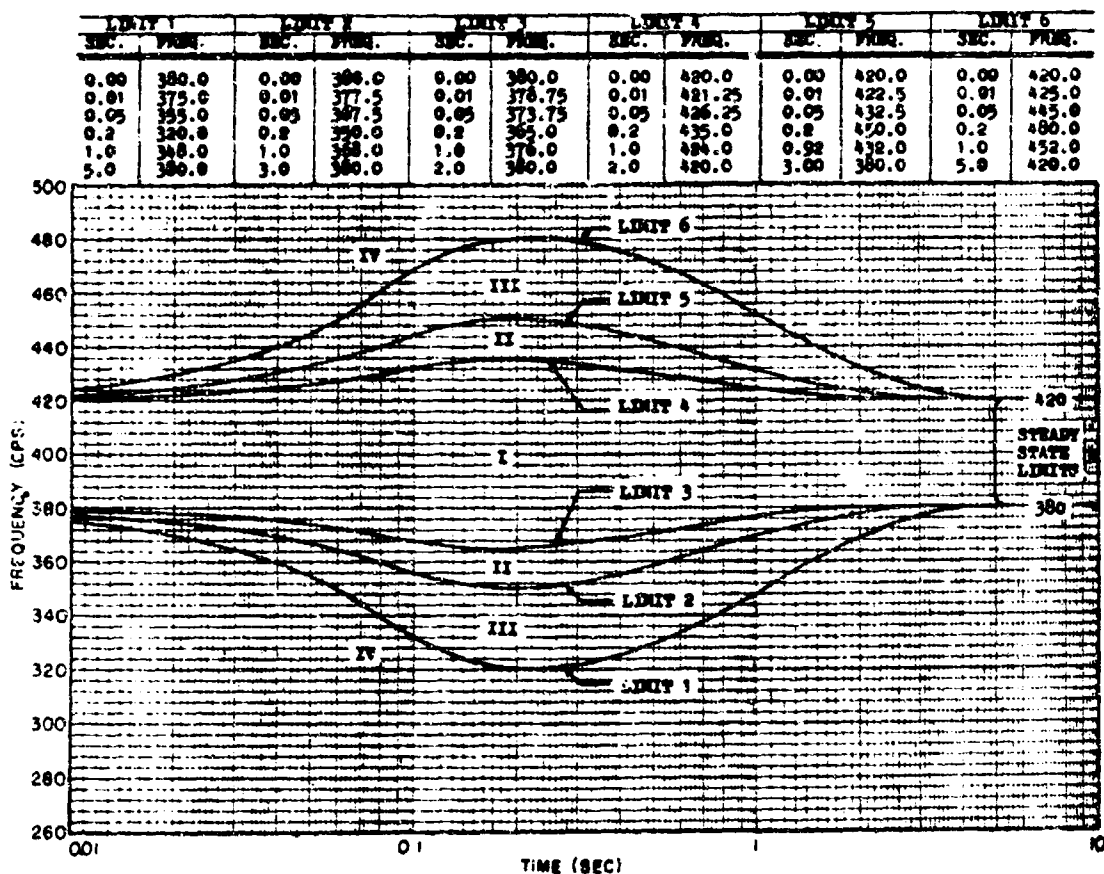


FIGURE 12 - Proposed Transient Frequency Limits

d. No frequency transient shall occur in area IV of figure 12, or have a rate of frequency change greater than 500 cps/s, during any electric system operation.

2. The following characteristics for frequency modulation:

a. Frequency modulation shall be within a ± 2 -cps band about a mean frequency during any one-minute of system operation. This band shall always be within the steady state frequency limits.

b. Any rate of frequency change due to frequency modulation shall not be greater than 13 cps/s.

3. The following requirements for utilization equipments:

a. During transients in area I of figure 12 or with rates of transient frequency change less than 125 cps/s, utilization equipment shall provide 100 percent performance and remain unaffected in reliability.

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

b. During transients in area II of figure 12, or with rates of transient frequency change between 125 and 250 cps/s, utilization equipment shall:

- (1) provide 100 percent performance unless the detail specification for a given utilization equipment defines specific regions and degrees of performance degradation,
- (2) remain safe,
- (3) recover to 100 percent performance automatically after system recovery from the transient when any degraded performance is allowed,
- (4) remain unaffected in reliability.

c. During transients in area III of figure 12, or with rates of transient frequency change between 250 and 500 cps/s, utilization equipment:

- (1) shall not be required to perform unless the equipment detail specification requires specific regions and degrees of performance,
- (2) may have momentary loss of function during the transient,
- (3) shall remain safe,
- (4) shall return to 100 percent performance automatically unless equipment detail specification permits manual reset after transient recovery,
- (5) recover with negligible effect on reliability.

d. During transients in area IV of figure 12 or with rates of transient frequency change above 500 cps/s, utilization equipment:

- (1) shall not be required to perform,
- (2) shall remain safe during and after the transient,
- (3) shall precipitate no adverse conditions except those caused by the natural loss of the equipment function,
- (4) may be affected in reliability.

REFERENCES

- (a) AIEE Transaction Paper 56-436, "Frequency Modulation and Load-Division Instability in 400-Cycle Aircraft Electric Systems" by Henry Oman of 14 Feb 1956
- (b) BUAER ltr Aer-EL-52 ser 91081 of 3 Jul 1953
- (c) Spec No. MIL-E-7894A(ASG), "Characteristics of Aircraft Electric Power" of 17 May 1955
- (d) Report No. NADC-EL-L5589, "Phase Report No. 1, Development of Definitions for Voltage Modulation," of 12 Dec 1955
- (e) "Electronics," Feb 1954, p 188, "Phase-Selective Detectors" by Curtis R. Schafer
- (f) Book "Theory and Application of Industrial Electronics" by John M. Cage and C. J. Baske, p 223, published 1951 by McGraw Hill Book Company
- (g) "Electronic Design," Jul 1954, p 30, "Laboratory Phase Comparator" by S. Feinstein

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

T A B L E I

ELECTRIC SYSTEM COMPATIBILITY RELATIONSHIPS FOR FREQUENCY TRANSIENTS BETWEEN AIRCRAFT ELECTRIC SYSTEMS AND UTILIZATION EQUIPMENTS

T A B L E I
ELECTRIC SYSTEM
COMPATIBILITY RELATIONSHIPS FOR FREQUENCY TRANSIENTS
BETWEEN AIRCRAFT ELECTRIC SYSTEMS AND UTILIZATION EQUIPMENTS

Area I	Area II	Area III	Area IV
(D) Normal load switching	(D) Bus switching	(D) Faults and recovery from faults	(F) No electric system operation in this area
(F) All flight conditions	(D) Synchronizing	(D) Reset after fault not seen by utilization equipment	
(F) Usual electric system operations required for aircraft function and mission	(D) Warmup	(F) Electric system operation in this area occurs rarely, perhaps once during any given aircraft flight	
(F) Occurrence under manual or automatic control	(F) Electric system operations required for function of electric system	(F) Occurrence not under any manual or automatic control	
	(F) Usually tied in to distinct aircraft operational conditions	(F) Recovery under automatic or manual control	
	(F) Electric system operations in this area occur occasionally, perhaps 3 times during one operational period		

Note: Functional - (F)
Design - (D)

T A B L E I I

UTILIZATION EQUIPMENT COMPATIBILITY RELATIONSHIPS FOR FREQUENCY TRANSIENTS BETWEEN AIRCRAFT ELECTRIC SYSTEMS AND UTILIZATION EQUIPMENTS

T A B L E I I
UTILIZATION EQUIPMENT
COMPATIBILITY RELATIONSHIPS FOR FREQUENCY TRANSIENTS
BETWEEN AIRCRAFT ELECTRIC SYSTEMS AND UTILIZATION EQUIPMENTS

Area I	Area II	Area III	Area IV
(F) 100% performance	(F) 100% performance (during transient) unless equipment detail spec defines allowance of regions and degrees of performance degradation	(F) No performance (during transient) unless equipment detail spec requires specific regions and degrees of performance to be maintained	(F) No performance required unless detail spec dictates applicable portions of Area IV and requires specific regions and degrees of equipment performance to be maintained
(F) No change in reliability	(F) Remain safe	(F) Remain safe	(F) Remain safe
	(F) Return to 100% performance automatically after recovery from transient	(F) Momentary loss of equipment functions not permitted to affect equipment performance after recovery from transient	(F) Return to performance not required unless called out in detail specification
	(F) No change in reliability	(F) Return to 100% performance automatically unless equipment detail spec allows manual reset of equipment	(F) To precipitate no adverse conditions except those caused by the natural loss of the equipment function
		(F) Negligible effect on reliability	(F) Reliability may be affected
			(D) Integral protection from overload

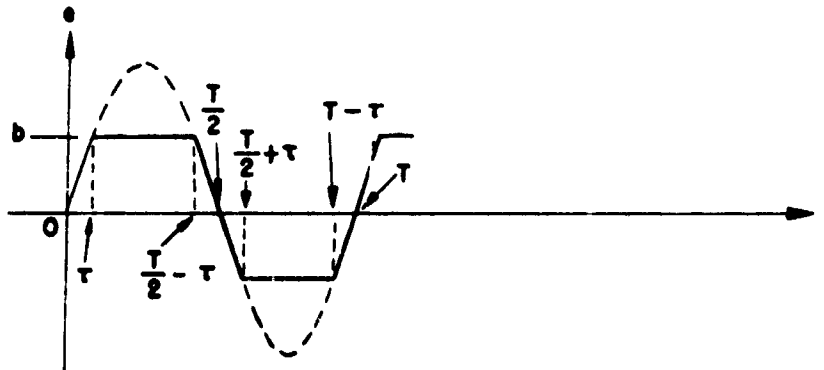
Note: Functional - (F)
Design - (D)

APPENDIX A

DERIVATIONS TO DETERMINE ERROR OF FUNDAMENTAL
COMPONENT OF A VARIABLE VOLTAGE SINE WAVE CLIPPED
AT A CONSTANT AMPLITUDE

By S. Wolin

- Given: E_m = Peak voltage of a variable voltage sine wave.
 b = Peak voltage at the constant clipping level.
 e = Variable voltage at any instant.
 T = Total period of a sine wave.
 τ = Time at which clipping starts.



$$(1) \quad e = E_m \sin \omega t, \quad \omega = \frac{2\pi}{T}, \quad \frac{1}{\omega} = \frac{T}{2\pi}.$$

$$(2) \quad b = E_m \sin \omega \tau$$

$$\text{also: } b = E_m \sin \omega \left(\frac{T}{2} - \tau \right).$$

$$(3) \quad \text{From (2) } \sin \omega \tau = \frac{b}{E_m}$$

$$\text{and } \omega \tau = \arcsin \left(\frac{b}{E_m} \right)$$

$$\text{therefore } \tau = \frac{T}{2\pi} \arcsin \left(\frac{b}{E_m} \right).$$

REPORT NO. NADC-EL-5675

- (4) General Fourier analysis of a periodic function is given by the expression:

$$e(t) = B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + \dots \\ + A_1 \sin \omega t + A_2 \sin 2\omega t + \dots$$

In this case $B_0 =$ zero since there is no d-c component. Also since in this case there are no even harmonics, $B_1 = 0, B_2 = 0, \dots$

Hence in this case:

$$e(t) = A_1 \sin \omega t + A_2 \sin 2\omega t + \dots + A_n \sin n\omega t,$$

$$\text{where } A_n = \frac{2}{T} \int_0^T e(t) \sin(n\omega t) dt.$$

- (5) To obtain the general expression for the Fourier coefficients of the discontinuous wave, the period 0 to T is divided into the intervals 0 to τ , τ to $\frac{T}{2} - \tau$, $\frac{T}{2} - \tau$ to $\frac{T}{2} + \tau$, $\frac{T}{2} + \tau$ to $T - \tau$ and $T - \tau$ to T.

$$A_n = \frac{2}{T} \left[\int_0^{\tau} E_m \sin \omega t (\sin n\omega t) dt \right. \\ + \int_{\tau}^{\frac{T}{2} - \tau} b \sin(n\omega t) dt \\ + \int_{\frac{T}{2} - \tau}^{\frac{T}{2} + \tau} E_m \sin \omega t (\sin n\omega t) dt \\ + \int_{\frac{T}{2} + \tau}^{T - \tau} (-b) \sin(n\omega t) dt \\ \left. + \int_{T - \tau}^T E_m \sin \omega t (\sin n\omega t) dt \right]$$

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

- (6) Since the fundamental component after clipping is the item of interest, $n = 1$ is used to derive the fundamental component, and thus:

$$A_1 = \frac{2}{T} \left[E_m \int_0^r \sin^2 \omega t \, dt \right. \\ + b \int_r^{\frac{T}{2} - r} \sin \omega t \, dt \\ + E_m \int_{\frac{T}{2} - r}^{\frac{T}{2} + r} \sin^2 \omega t \, dt \\ + (-b) \int_{\frac{T}{2} + r}^{T - r} \sin \omega t \, dt \\ \left. + E_m \int_{T - r}^T \sin^2 \omega t \, dt \right]$$

- (7) Integrating the expression, it becomes:

$$A_1 = \frac{2}{T} \left[\left(\frac{E_m r}{2} - \frac{E_m \sin 2\omega r}{4\omega} \right) \right. \\ + \left(\frac{2b \cos \omega r}{\omega} \right) \\ + \left(\frac{E_m r}{2} - \frac{E_m \sin 2\omega r}{4\omega} \right) \\ + \left(\frac{2b \cos \omega r}{\omega} \right) \\ \left. + \left(\frac{E_m r}{2} - \frac{E_m \sin 2\omega r}{4\omega} \right) \right]$$

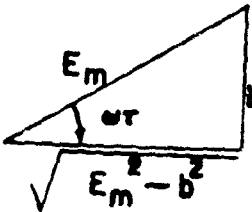
Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

- (8) Simplifying and collecting terms;

$$A_1 = \frac{2}{T} \left[2 E_m \tau - \frac{E_m \sin 2\omega\tau}{\omega} + \frac{4 b \cos \omega\tau}{\omega} \right]$$

- (9) From equation (2):

$$\cos \omega\tau = \frac{\sqrt{E_m^2 - b^2}}{E_m}$$


- (10) $\sin 2\omega\tau = 2 \sin \omega\tau \cos \omega\tau$; then substituting from equations (9) and (3):

$$\begin{aligned} \sin 2\omega\tau &= 2 \left(\frac{b}{E_m} \right) \left(\frac{\sqrt{E_m^2 - b^2}}{E_m} \right) \\ &= 2b \frac{\sqrt{E_m^2 - b^2}}{E_m^2} \end{aligned}$$

- (11) Substituting equations from (1), (3), (9), and (10) into (8) to remove the time function:

$$\begin{aligned} A_1 &= \frac{2}{T} \left[2 E_m \frac{T}{2\pi} \arcsin \left(\frac{b}{E_m} \right) - E_m \left(\frac{T}{2\pi} \right) \frac{2b \sqrt{E_m^2 - b^2}}{E_m^2} \right. \\ &\quad \left. + 4b \left(\frac{T}{2\pi} \right) \frac{\sqrt{E_m^2 - b^2}}{E_m} \right] \end{aligned}$$

- (12) A final expression for the amplitude of the fundamental frequency:

$$A_1 = \frac{2 E_m}{\pi} \arcsin \left(\frac{b}{E_m} \right) + \frac{2b \sqrt{E_m^2 - b^2}}{\pi E_m}$$

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5675

- (13) For the condition when clipping starts, b equals E_m and the expression reduces to:

$$A_1 = \frac{2 E_m}{\pi} \arcsin\left(\frac{E_m}{E_m}\right) + \frac{2 E_m \sqrt{E_m^2 - E_m^2}}{\pi E_m}$$

$$\text{and } A_1 = E_m$$

For the condition when E_m is infinitely large in relation to b (the clipping level), the expression reduces to $A_1 = 1.27 b$. Thus, when the starting point of E_m is large in reference to the level at which it is clipped, as it is reduced to the clipping level (b) there is a decrease of the fundamental component from 1.27 to 1. This constitutes an error as high as $\frac{0.27}{1.27}$ or 21.3 percent in the assumption that squaring a wave of varying amplitude maintains a constant output at the fundamental frequency.

Aeronautical Electronic and Electrical Laboratory

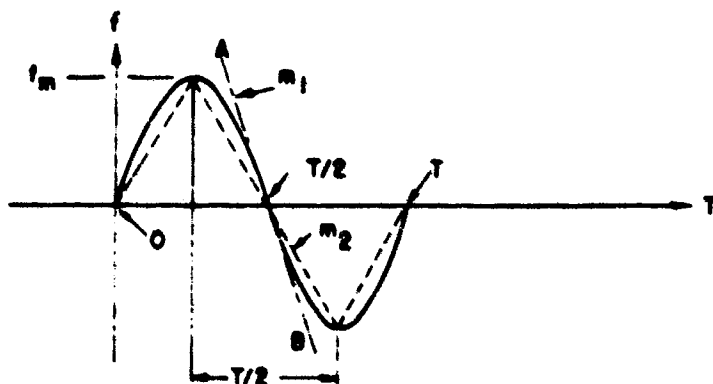
REPORT NO. NADC-EL-5675

APPENDIX B

DERIVATION TO DETERMINE THE RATIO OF SLOPE
OF A TRIANGULAR WAVE TO THE SLOPE OF A SINE WAVE
AT ITS POINT OF INFLECTION, BOTH WAVES
HAVING THE SAME TIME PERIOD AND MAXIMUM AMPLITUDE

By S. Wolin

- Given: f_m = maximum deviation in terms of frequency.
 f = variable frequency at any point.
 T = total time period of both the sine wave and the triangular wave.
 m_1 = slope of sine wave at the point of inflection.
 m_2 = slope of triangular wave.



- (1) $f = f_m \sin \omega t$, where $\omega = 2\pi f$ and $\omega = \frac{2\pi}{T}$
(2) Differentiating the equation in (1) with respect to t :

$$\frac{df}{dt} = \omega f_m \cos \omega t$$

- (3) Differentiating equation (2) with respect to t :

$$\frac{d^2f}{dt^2} = -\omega^2 f_m \sin \omega t$$

Aeronautical Electronic and Electrical Laboratory

REPORT NO. NADC-EL-5676

- (4) For the point of inflection along the t axis $\frac{d^2 i}{dt^2} = 0$

$$\therefore \omega^2 i_m \sin \omega t = 0.$$

From the above equation it may be shown that the point of inflection and maximum slope, along the t axis is $T/2$.

- (5) The slope at the point of inflection can be determined by substituting $t = T/2$ in equation (2):

$$\frac{di}{dt} = -\frac{2\pi}{T} i_m = m_1.$$

- (6) From the figure it is evident that the slope of the triangular wave going through the point $T/2$ is given by:

$$m_2 = -\frac{2 i_m}{T/2} = -\frac{4 i_m}{T}$$

- (7) The ratio of slopes is then:

$$\frac{m_1}{m_2} = \frac{\pi}{2} = 1.571.$$